UK INTERMEDIATE MATHEMATICAL CHALLENGE

THURSDAY 1ST FEBRUARY 2018

Organised by the **United Kingdom Mathematics Trust**
and supported by

Institute
and Faculty
of Actuaries

RULES AND GUIDELINES (to be read before starting)

1. Do not open the paper until the Invigilator tells you to do so.

2. Time allowed: **1 hour**.
    No answers, or personal details, may be entered after the allowed hour is over.

3. The use of rough paper is allowed; **calculators** and measuring instruments are **forbidden**.

4. Candidates in England and Wales must be in School Year 11 or below.
    Candidates in Scotland must be in S4 or below.
    Candidates in Northern Ireland must be in School Year 12 or below.

5. **Use B or HB pencil only**. Mark *at most one* of the options A, B, C, D, E on the
    Answer Sheet for each question. Do not mark more than one option.

6. *Do not expect to finish the whole paper in 1 hour.* Concentrate first on Questions 1-15.
    When you have checked your answers to these, have a go at some of the later questions.

7. Five marks are awarded for each correct answer to Questions 1-15.
    Six marks are awarded for each correct answer to Questions 16-25.
    Each incorrect answer to Questions 16-20 loses 1 mark.
    Each incorrect answer to Questions 21-25 loses 2 marks.

8. Your Answer Sheet will be read only by a *dumb machine*. **Do not write or doodle on the
    sheet except to mark your chosen options**. The machine 'sees' all black pencil markings
    even if they are in the wrong places. If you mark the sheet in the wrong place, or leave bits of
    rubber stuck to the page, the machine will 'see' a mark and interpret this mark in its own way.

9. The questions on this paper challenge you to **think**, not to guess. You get more marks,
    and more satisfaction, by doing one question carefully than by guessing lots of answers.
    The UK IMC is about solving interesting problems, not about lucky guessing.

1. Which of these is the sum of the cubes of two consecutive integers?

   A 4　　　　　　　B 9　　　　　　　C 16　　　　　　　D 25　　　　　　　E 36

2. How many of these four integers are prime?

   $$1 \quad 11 \quad 111 \quad 1111$$

   A 0　　　　　　　B 1　　　　　　　C 2　　　　　　　D 3　　　　　　　E 4

3. In September 2016 a polymer £5 note was introduced. The Bank of England issued 440 million of them.
   What is the total face value of all these notes?

   A £220 000 000　　　　　　　B £440 000 000　　　　　　　C £2 200 000 000

   　　　　　　D £4 400 000 000　　　　　　　E £22 000 000 000

4. A kite is made by joining two congruent isosceles triangles, as shown.
   What is the value of x?

   A 36　　　B 54　　　C 60　　　D 72　　　E 80

5. The adult human body has 206 bones. Each foot has 26 bones.
   Approximately what fraction of the number of bones in the human body is found in one foot?

   A $\dfrac{1}{6}$　　　　　　B $\dfrac{1}{8}$　　　　　　C $\dfrac{1}{10}$　　　　　　D $\dfrac{1}{12}$　　　　　　E $\dfrac{1}{20}$

6. In 2014, in Boston, Massachusetts, Eli Bishop set a world record for the greatest number of claps per minute. He achieved 1020 claps in one minute.
   How many claps is that per second?

   A 17　　　　　　　B 16.5　　　　　　　C 16　　　　　　　D 15.5　　　　　　　E 15

7. How many two-digit squares have the property that the product of their digits is also a square?

   A 0　　　　　　　B 1　　　　　　　C 2　　　　　　　D 3　　　　　　　E 4

8. The diagram shows a square of perimeter 20 cm inscribed inside a square of perimeter 28 cm.
   What is the area of the shaded triangle?

   A 6 cm^2　　B 7 cm^2　　C 8 cm^2　　D 9 cm^2　　E 10 cm^2

9. Which integer n satisfies $\dfrac{3}{10} < \dfrac{n}{20} < \dfrac{2}{5}$?

   A 3　　　　　　　B 4　　　　　　　C 5　　　　　　　D 6　　　　　　　E 7

10. Which of these integers cannot be expressed as the difference of two squares?

    A 5　　　　　　　B 7　　　　　　　C 8　　　　　　　D 9　　　　　　　E 10

11. The diagram shows a regular hexagon which has been divided into six regions by three of its diagonals. Two of these regions have been shaded. The total shaded area is 20 cm². What is the area of the hexagon?
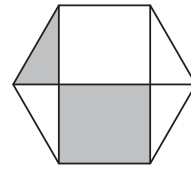
A 40 cm² B 48 cm² C 52 cm² D 54 cm² E 60 cm²

12. Someone has switched the numbers around on Harry's calculator!

The numbers should be in the positions shown in the left-hand diagram, but have been switched to the positions in the right-hand diagram.

Which of the following calculations will *not* give the correct answer when Harry uses his calculator?

A 79 × 97   B 78 × 98   C 147 × 369   D 123 × 321   E 159 × 951

13. The diagram shows a rhombus and two sizes of regular hexagon.

What is the ratio of the area of the smaller hexagon to the area of the larger hexagon?
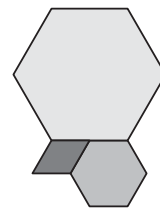
A 1 : 2   B 1 : 3   C 1 : 4   D 1 : 8   E 1 : 9

14. Which of these is equal to $\dfrac{10}{9} + \dfrac{9}{10}$?

A 1   B 2   C 2.0$\dot{1}$ D 2.$\dot{1}$ E 2.$\dot{2}$

15. How many of these four shapes could be the shape of the region where two triangles overlap?

equilateral triangle   square   regular pentagon   regular hexagon

A 0   B 1   C 2   D 3   E 4

16. The diagram shows a triangle with edges of length 3, 4 and 6.
A circle of radius 1 is drawn at each vertex of the triangle.
What is the total shaded area?

A 2π   B $\dfrac{9\pi}{4}$ C $\dfrac{5\pi}{2}$ D $\dfrac{11\pi}{4}$ E 3π
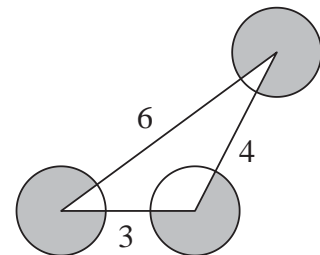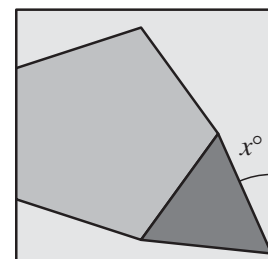

17. How many three-digit numbers are increased by 99 when their digits are reversed?
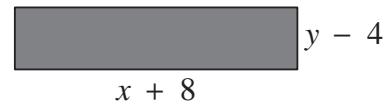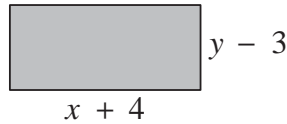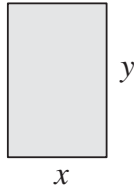
A 4   B 8   C 10   D 80   E 90

18. The diagram shows a regular pentagon and an equilateral triangle placed inside a square.
What is the value of x?

A 24   B 26   C 28   D 30   E 32

19. The three rectangles shown below all have the same area.
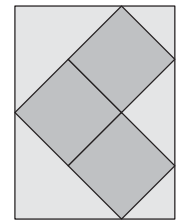

What is the value of $x + y$?

A 4   B 6   C 8   D 10   E 12

20. A particular integer is the smallest multiple of 72, each of whose digits is either 0 or 1. How many digits does this integer have?

A 4   B 6   C 8   D 10   E 12

21. For certain values of x, the list x, $x + 6$ and x^2 contains just two different numbers. How many such values of x are there?
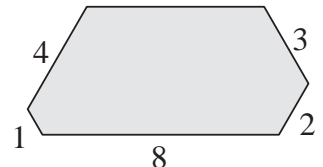
A 1   B 2   C 3   D 4   E 5

22. Three squares, with side-lengths 2, are placed together edge-to-edge to make an L-shape. The L-shape is placed inside a rectangle so that all five vertices of the L-shape lie on the rectangle, one of them at the midpoint of an edge, as shown.
What is the area of the rectangle?

A 16   B 18   C 20   D 22   E 24

23. The diagram shows a hexagon. All the interior angles of the hexagon are 120°. The lengths of some of the sides are indicated. What is the area of the hexagon?
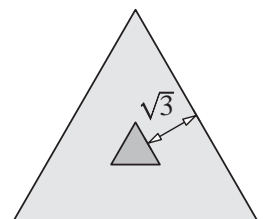
A $20\sqrt{3}$ B $21\sqrt{3}$ C $22\sqrt{3}$ D $23\sqrt{3}$ E $24\sqrt{3}$

24. A list of 5 positive integers has mean 5, mode 5, median 5 and range 5. How many such lists of 5 positive integers are there?

A 1   B 2   C 3   D 4   E 5

25. The diagram shows two equilateral triangles. The distance from each point of the smaller triangle to the nearest point of the larger triangle is $\sqrt{3}$, as shown.
What is the difference between the lengths of the edges of the two triangles?

A $2\sqrt{3}$ B $4\frac{1}{2}$ C $3\sqrt{3}$ D 6   E $4\sqrt{3}$